Analytical Investigations into Thermal Resistance of Diaphragm Wall Heat Exchangers

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ABSTRACT

Geothermal energy is a promising and sustainable source that can reduce current dependence on conventional fuels for thermal energy production. To exploit this source of energy complex thermo-active geostructures such as diaphragm wall heat exchangers are being investigated theoretically as well as experimentally. These geostructures are composed of concrete panels embedded with reinforcement cages fitted with absorber pipes. The thermal boundary surfaces are applied at surfaces representing the adjacent ground and the exposed concrete surface in addition to the pipe surface. An effective and optimised design for such geostructures is fundamental to efficient energy production. In the present study the thermal resistance of such complex thermo-active substructures are quantified using boundary collocation least squares method. To that end, steady state temperature distribution in a two dimensional cross section of a diaphragm wall heat exchanger is investigated. Design parameters including pipe and heat exchanger fluid specifications, pipe embedded section geometry and convective boundary conditions representing the fluid and the ground are used as model inputs. Thermal resistances are investigated for varying design parameters over a range of thermal properties of the ground and the concrete.

1. INTRODUCTION

To date, numerous investigations into performance of thermos-active piles have been carried out (Esen and Inalli (2009); Hepbasli (2003); Li et al. (2007); Lim et al. (2007)) while diaphragm wall heat exchangers (DWHE) are less researched, Rees (2016). Brandl (2006) studied heat transfer in diaphragm walls applied in three main pilot projects including a rehabilitation centre, a traffic tunnel, and metro stations in addition to further smaller projects. Adam and Markiewicz (2009) performed finite element simulations to calculate heating and cooling performance of absorber elements such as diaphragm walls at different absorber distances. They reported that with larger distances the thermal power will decrease as well as the installation cost tending to an optimum. However, it was emphasized that their results are only valid for the studied case and may vary significantly for other geothermal systems while the method is applicable to any geothermal energy system installed in foundation elements. Xia et al. (2012) made the first attempt to experimentally investigate heat transfer performance of DWHEs and through comparison with borehole heat exchangers (BHE) revealed heat transfer characteristics of DWHEs and the factors influencing heat exchange rate in such geostructures. Sun et al. (2013) developed two-dimensional heat transfer models for DWHEs according to the structural features, i.e. over and under the excavation line, based on which a design model for such energy geostructures was proposed. Their models showed good agreement with numerical solutions and measured data. Kürten et al. (2013) proposed a new numerical approach for the thermal analysis of DWHEs and verified their findings with laboratory tests. Bourne-Webb et al. (2016a) identified communalities and differences between the methods used for evaluating BHEs and energy geostructures. In another study Bourne-Webb et al. (2016b) conducted numerical analysis to establish the heat exchange mechanisms in DWHEs, and reported that the main mechanism is between the air-void and wall rather than the ground. Coletto and Sterpi (2016) used coupled thermo-mechanical analysis to study the heat transfer effects on the soil temperatures, the wall internal actions and the soil-structure interaction. Di Donna et al. (2016) used numerical simulation and statistical analysis to highlight the parameters governing energy efficiency in DWHEs. Soga and Rui (2016) summarised the current understanding on the performance of energy geostructures and discussed some design considerations. They suggested that more work is required to build confidence in the use of such substructure heat exchangers. Sterpi et al. (2017) investigated the energy performance as well as short and long term influence on the soil temperatures using finite element thermal analysis. Furthermore, they carried out finite element thermo-mechanical analysis to highlight the wall geotechnical and structural response. In their recent research Sterpi et al. (2018) have investigated various factors affecting the performance of DWHEs, including the layout of the exchanger pipe, the ratio between exposed and fully immersed parts of the wall, and the variable thermal condition at the excavation side, using finite volume analysis where a full-scale monitored diaphragm wall is used as reference. They have suggested an enhanced pipe arrangement that can improve the heat exchange rate by 15.8% for their studied case. In addition, their study showed that the energy performance can be improved by limiting the thermal interference between pipe branches circulating fluid at different temperatures, and by taking advantage of the fully immersed part of the wall, on both faces in direct contact with the soil. Rammal et al. (2018) reported that presence of significant groundwater flow and activating the whole length of diaphragm wall, both have positive impact on the heat exchange rate while the thermal performance of the walls are directly affected by the amount of thermal load. Barla et al. (2018) investigated the energy efficiency of DWHEs using finite element thermo-hydro coupled analyses together with the effects of the thermal activation on the surrounding soil. Furthermore, they employed finite difference thermo-mechanical analyses to study the mechanical effects induced by the thermal activation. They reported that horizontal pipe layout will maximise the heat exchange rates.

Evaluating thermal performance and efficiency of geothermal heat exchangers are largely dependent on assessing their thermal exchange rates. However, such assessments are not easy for complex geostructures and depend on various factors spanning from geometry specifications to thermal properties of the heat exchanger fluid, pipe, concrete and ground, etc. One simplified approach for such analysis is to formulate the “thermal resistance” of the geothermal system. To that end, it is necessary to study temperature distribution in these geostructures. In the present study we demonstrate a model to analytically quantify thermal resistances of DWHEs where design parameters are used as inputs for the model.

2. DWHE THERMAL RESISTANCE

Analysing heat transfer process in a substructure heat exchanger at shorter length or time scales is more appropriate using its thermal resistance. In a substructure heat exchanger thermal resistance attributes to the thermal resistance between the fluid in the embedded pipes and the heat exchanger wall and is a key performance characteristic. Thermal resistance, R, is related to steady state heat conduction rates, Q, using equation [1]:

$$R = -\frac{\Delta T}{Q} = \frac{1}{\lambda S} \qquad [1]$$

where ΔT is the temperature difference between the fluid in the pipe and the ground, i.e. the internal and external surfaces respectively, λ is thermal conductivity and S is the shape factor. Obviously, lower screen wall thermal resistance allows better system performance. There has been numerous methods proposed for such calculations for various types of substructure heat exchangers including the borehole heat exchangers, however, none are focused on analysing this characteristic for DWHEs. In the present study we present a method for calculating local screen wall thermal resistance using geometrical specifications, fluid properties and ground conditions.

The concept of thermal resistance is often visualised as a series sum of resistances. The total fluid-to-ground resistance (R_{Total}) is assumed to be comprised of three major parts: pipe resistance (R_{Pipe}), concrete resistance ($R_{Concrete}$), and ground resistance (R_{Ground}). The pipe resistance includes convective resistance of the fluid in the pipe (R_{Fluid}) and the conductive resistance of the pipe (R_{PC}). The total resistance can be expressed as shown in equation [2]:

$$R_{Total} = R_{Pipe} + R_{Concrete} + R_{Ground} = (R_{Fluid} + R_{PC}) + R_{Concrete} + R_{Ground} \qquad [2]$$

Once R_{Total} and R_{Ground} are known the DWHE thermal resistance, corresponding to the sum of R_{Pipe} and $R_{Concrete}$, can readily be calculated using equation [2]. R_{Ground} can simply be calculated using equation [3], where L is the length of the ground field:

$$R_{Ground} = \frac{L}{\lambda_g} \qquad [3]$$

where λ_g is thermal conductivity of the ground. The length of ground field, L, is selected based on the wall dimensions and the thermal responses obtained using a combination of dynamic thermal networks (DTN) model and finite volume analysis as reported by Shafagh and Rees (2018). Quantities of λ_g are obtained through experimental analysis. To calculate R_{Total} a more detailed mathematical model is required, details of which are provided in section 2.1.

2.1 Mathematical Modelling of R_{Total}

In the present research the total thermal resistance of the screen wall heat exchangers, R_{Total} in equation. [2], is calculated through the semi-analytical boundary collocation least squares method. The geometrical arrangement of the investigated problem, which comprises a row of equally spaces parallel hollow channels in an infinite medium, is shown in Fig. 1. Neglecting the side effects the problem represents a two-dimensional configuration of parallel pipes in a homogenous wall where the pipes are placed closer to one side, Fig. 1-(i). The wall has a thickness of 2b and the circular channels each has radius r_i, a separation distance of 2a, and an eccentricity of e. Due to the symmetry of the configuration the domain can be reduced to that existing about a single hole as shown in Fig. 1-(ii). The problem is then reduced to a plane region bounded by an annular inner boundary and a rectangular outer boundary where as a result part of the outer boundary becomes adiabatic. Remaining outer boundary and the inner circular surface are subjected to convective conditions. Ultimately, the domain can be further reduced to that in Fig. 1-(iii) for its symmetry along the wall thickness.

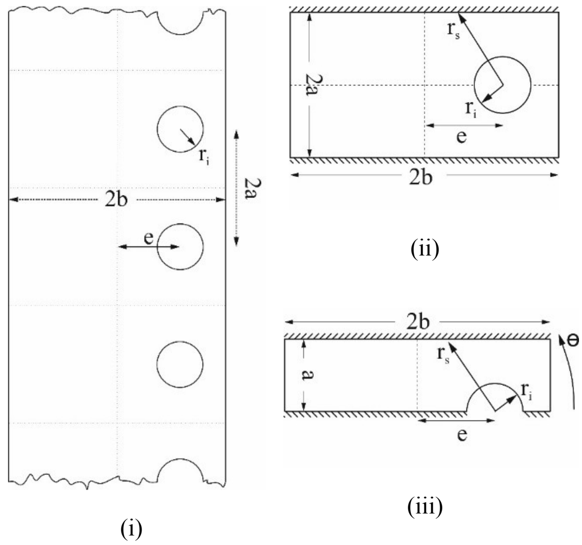

Figure 1: Schematic representation of the configuration showing the definition of the key geometric parameters: a, b, e, r_i, and r_s;

The equation that governs steady state temperature distribution in the domain illustrated in Fig. 1-(iii) is the Laplace’s equation in plane polar coordinates:

$$\frac{\partial^2 \Phi}{\partial R^2} + \frac{1}{R}\frac{\partial \Phi}{\partial R} + \frac{1}{R^2}\frac{\partial^2 \Phi}{\partial \theta^2} = 0 \quad [4]$$

where R and Φ are the dimensionless coordinate and temperature variables: $R = \frac{r}{\sqrt{a^2+b^2}}$ and $\Phi = \frac{T(r,\theta)-T(r_s,\theta)}{T(r_i,\theta)-T(r_s,\theta)}$. A linear combination of the trial functions that rigorously satisfies Laplace’s equation in the domain can be obtained by deploying the method of the separation of variables, when considering a circular domain, which can be simplified to that shown in equation [5]:

$$\Phi(R,\theta) = A + B\ln R + \sum_{n=1}^{\infty}(A_n R^n + B_n R^{-n})\cos(n\theta) \quad [5]$$

To simplify this equation the boundary conditions, corresponding to the inner pipe and outer rectangular surfaces, can be used. It should be noted that part of the outer rectangular surface, where $\tan^{-1}\left(\frac{a}{b+e}\right) \le \theta \le \pi - \tan^{-1}\left(\frac{a}{b-e}\right)$, is exposed to adiabatic conditions and therefore the normal gradient of the temperature along this section of the boundary is zero.

2.2 Boundary Conditions

Convective boundary conditions are applied to account for the fluid in the pipe and the ground surrounding the wall and therefore representative equations on the pipe surface and ground facing part of the rectangle surface will be as shown in equations [6] and [7] respectively:

$$\left.\frac{\partial \Phi}{\partial R}\right|_{R=R_{in}} - Bi_i(\Phi(R_{in},\theta) - 1) = 0 \quad [6]$$

$$\frac{R}{\sqrt{R^2+R'^2}}\left.\left(\frac{\partial \Phi}{\partial R} - \frac{R'}{R^2}\frac{\partial \Phi}{\partial \theta}\right)\right|_{R=\overline{R}} + Bi_o\Phi(\overline{R},\theta) = 0 \quad [7]$$

where $R_{in} = \frac{r_i}{\sqrt{a^2+b^2}}$, $\overline{R} = \frac{r_s}{\sqrt{a^2+b^2}}$ and $Bi_i = \frac{h_i\sqrt{a^2+b^2}}{k}$ and $Bi_o = \frac{h_o\sqrt{a^2+b^2}}{k}$ are the Biot numbers.

The equation governing the boundary condition on the adiabatic part of the rectangle section is:

$$\left.\left(\frac{\partial \Phi}{\partial R} - \frac{R'}{R^2}\frac{\partial \Phi}{\partial \theta}\right)\right|_{R=\overline{R}} = 0 \quad [8]$$

The appropriate heat transfer coefficients for the fluid in the pipe, i.e. h_{pi}, is determined from a convection correlation, which provides the Nusselt number:

$$h_{Pi} = \frac{Nu_{pi}\lambda_f}{2r_i} \quad [9]$$

where Nu_{pi} is the Nusselt number, λ_f is thermal conductivity of the fluid, and r_i is the internal diameter of the pipe.

A range of correlations to calculate the Nusselt number are available depending on flow regime, aspect ratio, and thermal boundary conditions. We will only give a single correlation for fully developed turbulent flow, generally considered to be one of the most accurate correlations. The Gnielinski correlation covers turbulent flows down to transition-to-laminar conditions:

$$Nu_{pi} = 0.023Re^{4/5}Pr^{0.4} \quad [10]$$

where *Re* and *Pr* are Reynolds and Prandtl numbers respectively. Prandtl number can be obtained using equation [11]:

$$Pr = \frac{C_P\mu}{\lambda_f} \quad [11]$$

where C_p is the specific heat and μ is the dynamic viscosity of the fluid.

By applying the boundary conditions in equation [5] two equations as follows are obtained:

$$A_0\left(\frac{1}{\sqrt{\overline{R}^2+\overline{R}'^2}} + \frac{Bi_o}{Bi_iR_{in}} + Bi_o\ln\frac{\overline{R}}{R_{in}}\right) + \sum_{n=1}^{\infty}A_n\left(\left(\frac{n}{\sqrt{\overline{R}^2+\overline{R}'^2}}\left(\overline{R}^n - \left(\frac{nR_{in}{}^{n-1}-Bi_iR_{in}{}^n}{nR_{in}{}^{-n-1}+Bi_iR_{in}{}^{-n}}\right)\overline{R}^{-n}\right) + Bi_o\left(\overline{R}^n + \left(\frac{nR_{in}{}^{n-1}-Bi_iR_{in}{}^n}{nR_{in}{}^{-n-1}+Bi_iR_{in}{}^{-n}}\right)\overline{R}^{-n}\right)\right)\cos(n\theta) + \left(\frac{n\overline{R}'}{\sqrt{\overline{R}^2+\overline{R}'^2}}\left(\overline{R}^{n-1} + \left(\frac{nR_{in}{}^{n-1}-Bi_iR_{in}{}^n}{nR_{in}{}^{-n-1}+Bi_iR_{in}{}^{-n}}\right)\overline{R}^{-n-1}\right)\right)\sin(n\theta)\right) = -Bi_o$$

[12]

$$\frac{A_0}{\overline{R}} + \sum_{n=1}^{\infty} nA_n\left(\left(\overline{R}^{n-1} - \left(\frac{nR_{in}^{\,n-1}-Bi_iR_{in}^{\,n}}{nR_{in}^{\,-n-1}+Bi_iR_{in}^{\,-n}}\right)\overline{R}^{-n-1}\right)\cos(n\theta) + \overline{R}'\left(\overline{R}^{n-2} + \left(\frac{nR_{in}^{\,n-1}-Bi_iR_{in}^{\,n}}{nR_{in}^{\,-n-1}+Bi_iR_{in}^{\,-n}}\right)\overline{R}^{-n-2}\right)\sin(n\theta)\right) = 0 \quad [13]$$

where equation [12] should be applied to ground facing part of the outer boundary and equation [13] to the adiabatic part. By solving these equations for A_0 the shape factor values as shown in equation [1] for the 2D model of DWHE can be quantified:

$$S = \int_S \frac{\partial \Phi}{\partial n} dS = -\int_0^{2\pi} \frac{\partial \overline{T}}{\partial R} R d\theta = -2\pi A_0 \quad [14]$$

Thermal resistance equals the inverse of the product of the shape factor and thermal conductivity of the medium conducting heat. To derive the values of A_0 one simple approach is applying boundary collocation least squares method (BCLSM) (Kolodziej and Strek (2000)). Boundary collocation method is an inherently mesh-less numerical approach proposed by Frazer et al. (1937) and greatly improved by Bickley (1941) by using it along with least squares technique and the Galerkin method to solve the unsteady heat condition problems. Through such methods the approximate solutions are forced on selected number of collocation points on the boundaries to satisfy the governing conditions. Using the least squares method minimizes the residual and increases the precision of the solutions obtained. In using a least squares approach an overdetermined system of linear equations can be used so that the number of collocation nodes, *M*, can exceed the number of unknowns, *N*. The coordinates of the collocation points are given by a combination of the angle θ and the distance r_s where r_s is a function of the angle θ.

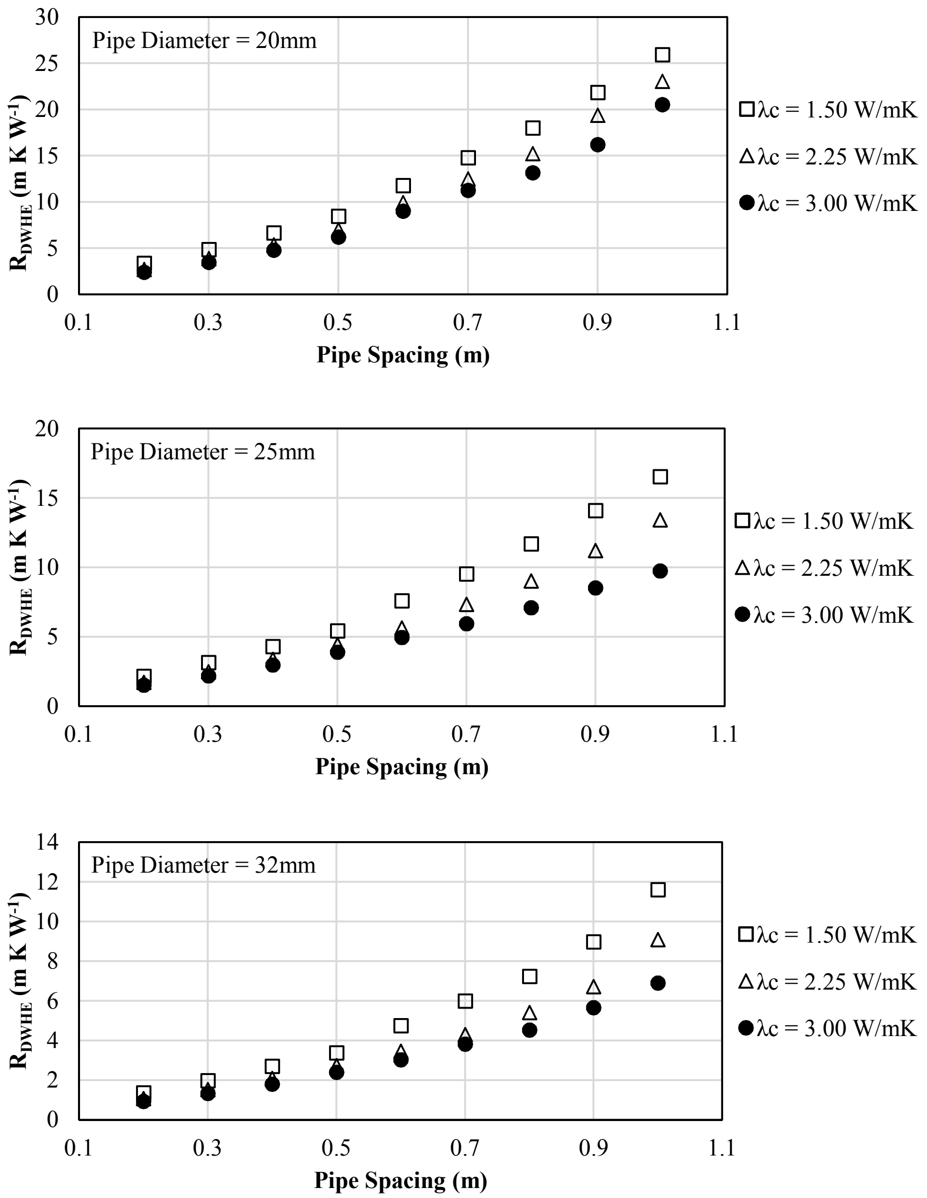

Figure 2: Variations of screen wall thermal resistance with various governing parameters.

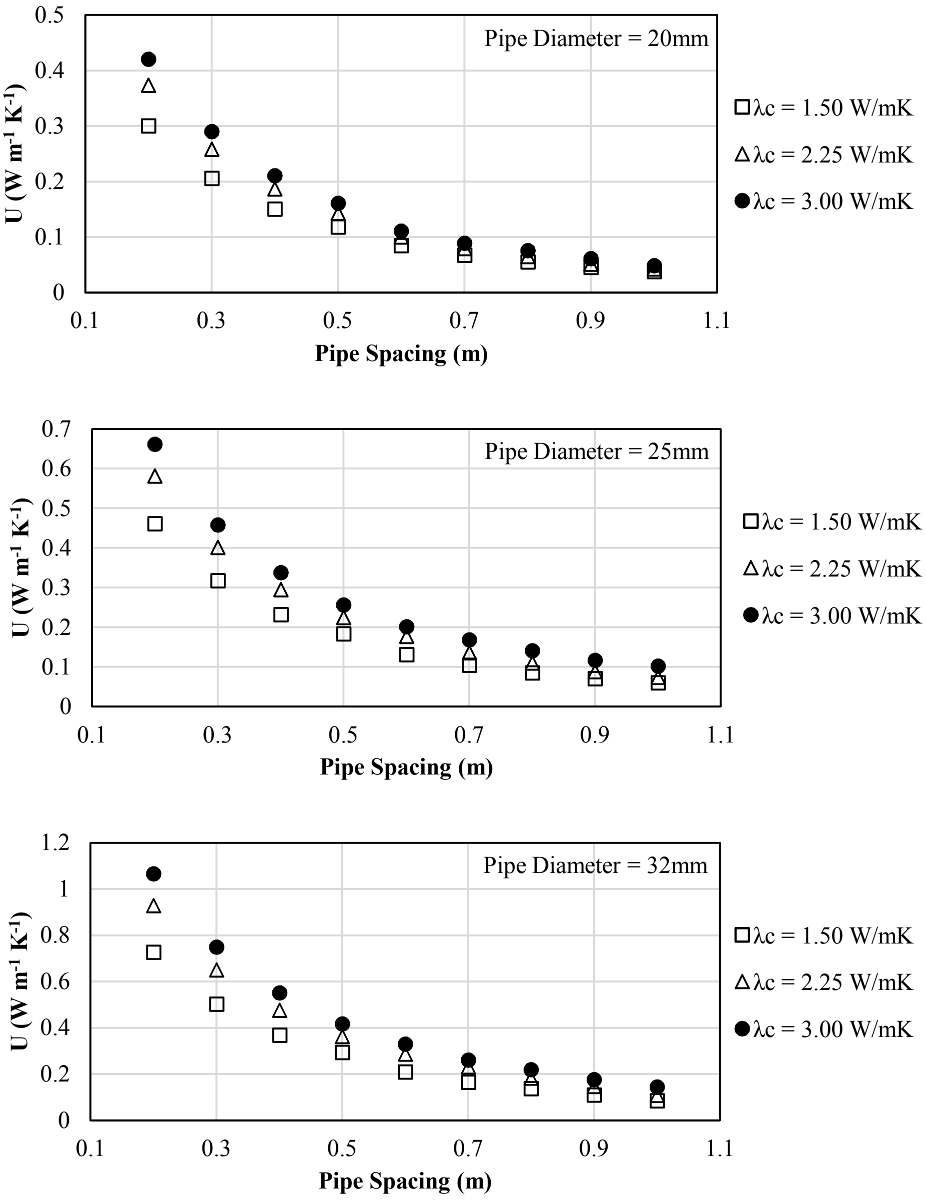

Figure 3: Variations of screen wall conductance with various governing parameters.

3. RESULTS & DISCUSSIONS

Sizing of ground source heat exchangers and hence the installation cost is affected significantly by the thermal resistance of the system. Fig. 2 and Fig. 3 present comparisons of DWHE thermal resistance and conductance, as introduced in equation [2], calculated using the model described in the previous section. These variations are demonstrate against variations in pipe spacing which is an important design parameter in DWHEs. These resistances are calculated for three pipe sizes: 20 mm, 25 mm, and 32 mm diameters and three concrete conductivities: 1.50 $kJkg^{-1}K^{-1}$, 2.25 $kJkg^{-1}K^{-1}$, and 3.00 $kJkg^{-1}K^{-1}$. The gap thickness area, which defines the interior between the pipe and wall surfaces, along with the boundary circumference are used to express the thermal resistance of the system for unit length of the concrete section.

It has been shown that with increase in pipe spacing thermal resistance of the system is increasing. As expressed in equation [1] it can be seen that increasing thermal conductivity of the concrete will decrease thermal resistance values. Moreover, at smaller pipe spacings, variations in thermal conductivity of the concrete has smaller impact on the thermal resistance of the system.

4. CONCLUSIONS

Evaluation of thermal resistances for a 2D model of the screen wall heat exchanger has been investigated using the semi-analytical boundary collocation least squares method. The results are studied for a range of pipe spacing and ground conductivities. The solutions proposed here for calculations of the thermal resistances are considerably less computationally burdensome than numerical methods and so can be implemented as a convenient design tool where results can be obtained directly from the geometric parameters and choice of boundary conditions with acceptable accuracy for many design purposes.

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